

Exercise 54

Find an equation of the tangent line to the curve at the given point.

$$y = xe^{-x^2}, \quad (0, 0)$$

Solution

A point on the tangent line is known, so all that's needed is its slope. Take a derivative of the given function

$$\begin{aligned} y' &= \frac{d}{dx}(xe^{-x^2}) = \left[\frac{d}{dx}(x) \right] e^{-x^2} + x \left[\frac{d}{dx}(e^{-x^2}) \right] = (1)e^{-x^2} + x \left[e^{-x^2} \cdot \frac{d}{dx}(-x^2) \right] \\ &= e^{-x^2} + x[e^{-x^2} \cdot (-2x)] \\ &= (1 - 2x^2)e^{-x^2} \end{aligned}$$

and evaluate it at $x = 0$.

$$y'(0) = (1 - 0)e^0 = 1$$

Therefore, the equation of the tangent line to $y = xe^{-x^2}$ at $(0, 0)$ is

$$y - 0 = 1(x - 0).$$

Below is a graph showing the function and the tangent line.

